

# Assessing Pre-service Mathematics Teachers' Professional Knowledge Through Lesson Planning

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**Abstract:** Recognised as a fundamental teaching competence, lesson planning quality is crucial for preservice teachers. This study aimed to develop and explore an innovative method for assessing the lesson plans of 33 Chinese preservice secondary mathematics teachers (PSMTs), using the Mathematical Knowledge for Teaching (MKT) framework. Participants submitted a mind map of lesson planning in general, and a written lesson plan on the topic of algebraic expressions, which were evaluated by 11 expert teachers using the CJ approach, supplemented by feedback collected via a questionnaire. Preliminary findings indicate a key imbalance: PSMTs demonstrated greater strength in Subject Matter Knowledge (SMK) compared to their Pedagogical Content Knowledge (PCK). The study's results provide actionable suggestions from expert teachers, offering practical directions for strengthening PSMTs' lesson-planning competence and professional growth.

**Keywords:** Lesson plans; pre-service teachers; Mathematics knowledge for teaching; Secondary; China

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## 1. Introduction

Lesson planning lies at the heart of effective teaching. It enables teachers to transform ideas about good teaching into purposeful classroom practice and is widely recognised as a fundamental component of teacher education across subjects and countries<sup>[1]</sup>. Through lesson planning, teachers anticipate learning processes, organise instructional activities, and align teaching with curricular goals and assessments, thereby shaping both the quality and direction of classroom learning.

In the broader literature on teacher professionalism, teacher competence is generally defined as the ability to perform professional tasks effectively, while teacher competency includes the underlying knowledge, skills, values, and dispositions that make this possible<sup>[2]</sup>. From this perspective, lesson planning represents a central expression of teacher competence. Effective planning requires teachers to make critical decisions about what to teach, how to teach it, how to assess learning, and how to evaluate instructional success. Thus, it represents a complex and reflective process that brings together multiple dimensions of professional knowledge.

Despite its acknowledged importance, many pre-service teachers continue to struggle with lesson planning. Studies have shown that novice teachers often adopt teacher-centred approaches, rely heavily on textbook structures, and rarely connect lesson content to students' prior knowledge or real-life experiences. Explanations and examples are frequently presented before students have had opportunities to explore mathematical ideas independently. Moreover, some pre-service teachers misconceive lesson planning as a mere procedural outline of curricular content. Ulusoy et al. (2023)<sup>[3]</sup> found that

only about 30% of pre-service teachers designed their own lesson materials, while most relied on online portals and ready-made workbooks, suggesting limited autonomy and reflective engagement in planning.

In China, lesson planning has a long-standing and central role in teacher education, lesson observation, and professional collaboration<sup>[4]</sup>. Crucially, lesson planning serves as the cornerstone of the Lesson Study process, a collaborative model that involves teachers in jointly designing, enacting, and reflecting on lessons to improve classroom practice<sup>[5]</sup>. In Lesson Study, planning is not simply preparatory; it is the intellectual core of professional learning, where teachers deliberate over content sequencing, anticipate student thinking, and design opportunities for inquiry and discussion. Related initiatives, including the “same lesson, different design” approach<sup>[6]</sup> and teacher competence competitions<sup>[7]</sup>, further underscore the centrality of lesson planning in teachers’ ongoing professional development and pedagogical refinement.

Nevertheless, research also points to challenges, such as the confusion between lesson planning and the mere adaptation of curriculum resources like slides or textbooks<sup>[8]</sup>. Although teaching rarely proceeds smoothly without careful planning, the quality of lesson plans is seldom evaluated systematically. This gap underpins the present study, which explores lesson planning as a productive window into teachers’ pedagogical and mathematical knowledge.

## 2. Literature review

A range of frameworks has been used to conceptualise teacher knowledge. Among these, the Mathematical Quality of Instruction (MQI) framework<sup>[9]</sup> focuses on teachers’ use of mathematical knowledge during real-time classroom interactions. While valuable for analysing live teaching, it is less suitable for examining written artefacts such as lesson plans. In contrast, Danielson’s Framework for Teaching<sup>[10]</sup> offers a broad and generalisable model for evaluating professional practice across subjects. However, its generic structure lacks the disciplinary specificity required to capture the unique reasoning processes involved in teaching mathematics.

To address this limitation, this study draws on the Mathematical Knowledge for Teaching (MKT) framework developed by Ball et al. (2008)<sup>[11]</sup>. The MKT framework initially expanded Shulman’s concept of pedagogical content knowledge (PCK), expanding it to distinguish more precisely between the types of knowledge teachers use in mathematics classrooms<sup>[12]</sup>. Through extensive empirical research involving more than 1,500 K–5 teachers, Ball et al. (2008) identified six interrelated subdomains that describe both the mathematical and pedagogical dimensions of teaching<sup>[11]</sup>.

Building on this work, Sapkota et al. (2024)<sup>[13]</sup> adapted the MKT framework into the context of lesson planning, illustrating how the MKT subdomains can be used to analyse teachers’ planning decisions about content, pedagogy, and student learning.

MKT provides a lens for examining how teachers draw upon mathematical and pedagogical knowledge when designing instruction. Building on this framework, our study investigates pre-service mathematics teachers’ lesson planning competence, and addresses two key research questions:

- (1) What is the nature of pre-service mathematics teachers’ understanding of lesson planning?
- (2) What are their strengths and weaknesses in lesson planning by the end of their training programme?

**Table 1.** Definitions and examples of six subdomains of mathematical knowledge for teaching (MKT)

MKT subdomains	Brief descriptions
Common content knowledge (CCK)	The general mathematical knowledge required to solve mathematics problems
Horizon content knowledge (HCK)	The knowledge of core disciplinary values and major structures of the discipline
Specialised content knowledge (SCK)	Specific mathematical knowledge in nuanced ways that include mathematical reasoning and multiple mathematical representations
Knowledge of content and teaching (KCT)	The knowledge of effective teaching strategies for teaching mathematics
Knowledge of content and students (KCS)	The understanding of students’ mathematical conceptions and reasoning
Knowledge of content and curriculum (KCC)	Knowledge of learning goals and horizontal and vertical organisations of mathematics across grade levels

Sources: Sapkota et al. (2024) presentation of MKT framework

### 3. Methodology

This study employed the Comparative Judgement (CJ) approach to assess pre-service teachers' lesson planning competence. CJ has been shown to be highly reliable for assessing skills in a range of contexts <sup>[14,15]</sup>, supported by the multiple judgments made on scripts. As Jones et al. pointed out, "One potential contribution that CJ might offer education is its suitability for assessing nebulous constructs that are deemed important, but which are difficult to specify comprehensively in mark schemes" <sup>[16]</sup>. Things that are important but difficult to define or capture through traditional rubrics, such as creativity, reasoning, or in our research, lesson-planning competence.

33 pre-service secondary mathematics teachers in their final year of study at a comprehensive university drew mind maps and unstructured written lesson plans on the topic of "algebraic expression". Their work was then assessed by 11 expert teachers. They compared pairs of lesson plans and judged which demonstrated better mathematical and pedagogical quality. Multiple comparisons were statistically modelled to generate a ranked scale of performance. The resulting scale achieved a separation reliability coefficient of 0.75, indicating strong agreement among expert judges.

A deductive content analysis approach was employed to examine pre-service teachers' lesson plans, following the procedures outlined by Bengtsson (2016) <sup>[17]</sup>. This approach was chosen because it allows researchers to apply an existing theoretical framework, in this case, the Mathematical Knowledge for Teaching (MKT) framework to guide the interpretation of textual data. The analysis focused on the manifest content, that is, the explicit and surface meanings of what had been written in the lesson plans.

### 4. Results

#### 4.1. Preservice teachers' understanding of mathematical knowledge for teaching

The study began by analysing the frequency of Mathematical Knowledge for Teaching (MKT) subdomains identified across all lesson plans. A total of 51 instances were recorded (**Table 2**). While the original MKT framework includes six subdomains, two additional categories, namely Knowledge about Students and Knowledge about Assessment, were introduced. This modification was necessary because six lesson plans could not be classified within the existing categories and were initially coded as General Pedagogical Knowledge.

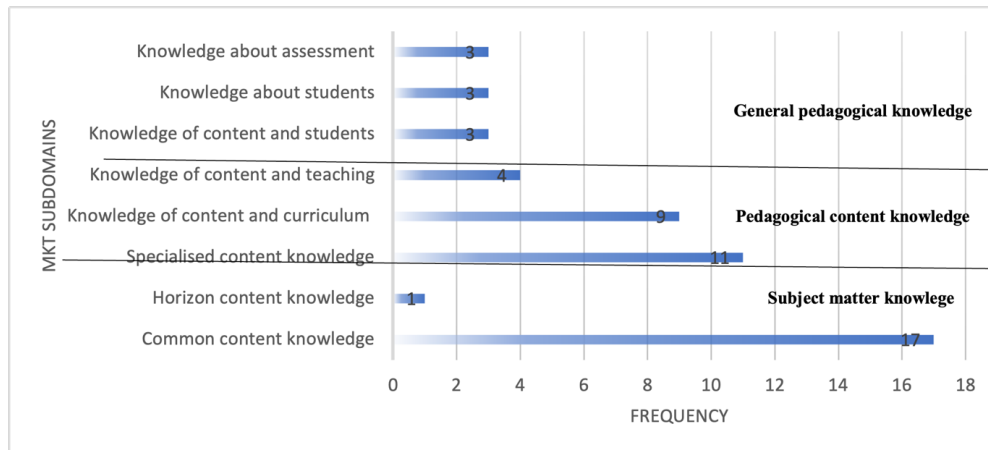
A closer analysis justified the inclusion of these new subdomains. Three lesson plans indicated that preservice teachers were prepared to assess students' thinking, although they did not employ explicit mathematical tasks to evaluate students' learning. Since this dimension was not represented in the original MKT framework, the subdomain Knowledge about Assessment was created to capture teachers' awareness of how to evaluate students' understanding. The remaining three lesson plans focused on knowledge about students, particularly in differentiating learning abilities, interests, and knowledge bases. These were coded accordingly. The final adapted framework therefore, consisted of eight subdomains (**Table 2**).

**Table 2.** Mathematical knowledge for teaching subdomains identified in written plans

MKT subdomains	Subject matter knowledge			Pedagogical content knowledge			General pedagogical knowledge	
	Common content knowledge	Horizon content knowledge	Specialized content knowledge	Knowledge of content and curriculum	Knowledge of content and teaching	Knowledge of content and students	Knowledge about students	Knowledge about assessment
Frequency (51)	17	1	11	9	4	3	3	3

Note: Frequencies represent coded instances of MKT subdomains across 33 lesson plans.

As shown in **Figure 1**, the three most frequently evidenced MKT subdomains were Common content knowledge (CCK), Special content knowledge (SCK), and Knowledge of content and curriculum (KCC). The first two fall under subject matter knowledge, suggesting that Chinese preservice teachers' lesson plans reflected more subject matter knowledge than pedagogical content knowledge. This is supported by the counts, with a total of 29 instances coded as subject matter knowledge and 16 instances as pedagogical content knowledge.



**Figure 1.** Mathematical knowledge for teaching (MKT) subdomains in written plans. This figure summarises the frequency of MKT subdomains identified in Chinese preservice mathematics teachers' lesson plans.

Common content knowledge (CCK) emerged as the most prominent subdomain. It was identified when the preservice teachers defined concepts, unpacked mathematical ideas, or presented maths problems for students to solve. Among these, solving maths problems was the most common. For instance, one plan introduced the concept of the system of linear equations in two variables, through a real-life scenario:

“Xiaoming’s mother bought 3 bottles of milk and 2 packs of cookies for 28 yuan. Xiaohong’s mother bought 2 bottles of milk and 3 packs of cookies for 27 yuan. How much does one bottle of milk cost, and how much does one pack of cookies cost?”

Special content knowledge (SCK) was the second most observed subdomain. It was evident in lesson plans that employed multiple representations to explore and reason about mathematical concepts. For instance, in a lesson on algebraic expressions, several representational forms were used to help students grasp the idea of letters as numbers (**Figure 2**). Through contextualised problems, teachers aimed to support students in comparing and contrasting how letters can symbolise unknown quantities.

A list of questions ordered in a particular sequence:

1. The price of rice is yuan per kilogram, and the price of olive oil is yuan per kilogram. Buying 10 kilograms of rice, 2 kilograms of olive oil would cost?
2. A rectangular garden has side lengths and . Find its area.

**Figure 2.** An example from the Chinese lesson plan.

Finally, the third most frequent subdomain was Knowledge of content and curriculum (KCC). This subdomain was typically observed when pre-service teachers identified connections between prior and future learning, illustrating how lessons fit within broader curricular sequences.

## 4.2. Strengths and areas for future development

Expert evaluations revealed that pre-service teachers demonstrated stronger Subject Matter Knowledge than Pedagogical Content Knowledge. Their strengths were most evident in Common Content Knowledge and Specialised Content

Knowledge, where they showed competence in unpacking mathematical reasoning and representing ideas in varied ways.

However, several areas for professional growth were identified. Weaknesses in Horizon Content Knowledge (HCK) and Knowledge of Content and Curriculum (KCC) suggest that trainees require further support in connecting individual lessons to a coherent progression of mathematical learning and in recognising the overarching structures of mathematical ideas. These aspects are essential for developing conceptual continuity across topics.

Another key area for development concerns the Knowledge of content and Students. Many pre-service teachers struggled in anticipating student misconceptions or leveraging learners' perspectives to inform lesson design. Strengthening these domains may enhance teachers' ability to design lessons that are both mathematically rigorous and responsive to students' needs.

## 5. Discussion and conclusion

This study demonstrates the potential of the Mathematical Knowledge for Teaching (MKT) framework as an analytical tool for examining pre-service teachers' lesson planning competence. While the framework proved useful for identifying different dimensions of mathematical and pedagogical knowledge represented in lesson plans, the findings suggest that the MKT framework requires further refinement to capture the nuanced ways in which pre-service teachers conceptualise and enact their knowledge of teaching mathematics.

Overall, pre-service teachers' understanding of MKT was found to focus mainly on common content knowledge (CCK) and specialised content knowledge (SCK), reflecting a problem-solving orientation<sup>[18]</sup> in their lesson planning. Their emphasis on core mathematical ideas and multiple solution strategies suggests an awareness of how problem-solving can support conceptual understanding, even though other MKT domains, such as horizon content knowledge (HCK) and knowledge of content and teaching (KCT) were less evident.

Among the Chinese preservice mathematics teachers, strengths were particularly evident in Common Content Knowledge (CCK), and Specialised Content Knowledge (SCK), reflecting a solid grasp of mathematical structures and reasoning. These strengths illustrate their growing disciplinary understanding and ability to represent mathematical ideas in meaningful ways. Conversely, areas such as Horizon Content Knowledge (HCK), Knowledge of Content and Students (KCS), and Knowledge of Content and Curriculum (KCC) emerged as priorities for further development. These domains are critical for supporting teachers in making curricular connections, anticipating students' learning progressions, and aligning pedagogical decisions with broader curricular goals.

In conclusion, future research should explore the longitudinal development of lesson planning competence as pre-service teachers transition into early-career practice, examining how these knowledge domains evolve and interact over time, providing valuable insights for designing teacher education programmes that cultivate balanced and reflective planning competence.

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