RESEARCH ARTICLE

Weibel instability in weakly relativistic laser fusion plasma

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Abstract: In this work, we used the MATLAB code to calculate the growth rate of the Weibel instability (WI) in weakly relativistic parameters of laser-plasma interaction in the inertial thermonuclear fusion. In this study, the basic equation is the relativistic Fokker-Planck equation (FPE). However in our paper, the distribution function is not supposed but it is calculated from FPE by considering the fusion plasma heating by the laser source and the collisions term which corresponds to the laser fusion experiments. The main obtained result is a decrease in the spectral range of Weibel unstable modes. This decreasing is accompanied by a reduction of two orders in the growth rate spectrum of instability, this spectrum presents a maximum, which can be in terpreted by the competition between the loss effects (collisions and Landau damping) and the inverse bremsstrahlung absorption (IBA) with parameters of laser-plasma interactions.

Keywords: relativistic Weibel instability; laser fusion plasma, static magnetic field; stabilization; Relativistic laser plasma interaction.

1. Introduction

Weibel instability [1] is a micro instability. It corresponds to the excitation of electromagnetic modes in plasmas characterized by temperature anisotropy. In a microscopic way, this corresponds to plasma described by an anisotropic distribution function in velocity space. The temperature anisotropy can be generated in plasma by different mechanisms, specifically the heat transport, the expansion of the plasma, and the inverse bremsstrahlung absorption [2]. We aim in this work to investigate the WI due to inverse bremsstrahlung absorption taking into account the stabilization effect due to the coupling of the self-generated magnetic field by the WI with the laser wave field in the relativistic regime, these needs to derive the dispersion relation of low-frequency electromagnetic Weibel modes in plasma heated by a laser pulse. The basic equation in this investigation is the relativistic FPE [3].

It results highlight new terms in the dispersion relation due to the coupling between the laser electric field, and the resulting magnetic field by the WI. These terms contribute to the instability and the convection of Weibel modes. We consider inhomogeneous plasma in interaction with a high frequency and low magnitude laser field. We calculate the distribution function from the anisotropic FPE. For this, we use the method of separation of time scales and the iterative method. After, we solve the linear part of the FPE associated with the disruption of the distribution function and establish the dispersion relation of the Weibel modes. Solving the dispersion relation leads to the calculation of the instability growth rate.

The present work is organized as follows: in section 1, we present our theoretical model, which is the FPE. In section 2, we calculate the distribution function. In section 3, we present an analysis of WI. Finally in section 4 and 5 we present a discussion of results and a brief conclusion summarizing our main results is given.

2. Theoretical model

doi: 10.18063/eoaa.v2i2.945

To describe fully ionized plasma where interactions between particles are dominated by the Coulomb interactions, it is judicious to use the FPE given in the Ref [4]. For electrons, it is written in the laboratory frame as:

$$\frac{\partial f}{\partial t}+\frac{\vec{p}}{m_e\gamma_l}.\frac{\partial f}{\partial \vec{r}}-e\left(\vec{E}+\frac{\vec{p}}{m_e\gamma_l}\times\vec{B}\right).\frac{\partial f}{\partial \vec{p}}=C_{ei}(f)+C_{ee}(f) \quad (1)$$

Where $f=f(\vec{r},\vec{p},t)$ is the electrons distribution functions, $\gamma_l=(1+\frac{p^2}{m^2c^2})^{1/2}$ is the relativistic Lorentz factor, $p=m\gamma_l v$ is the quantity of movement, m_e is the electron mass and e is the elementary charge.

$C_{ee}(f)$ and $C_{ei}(f)$ mean respectively is the electron-electron and electron-ion collision [5].

$\vec{E}$ and $\vec{B}$ are respectively the electric and the magnetic fields present in the plasma.written as: $\vec{E}=\vec{E}_h+\vec{E}_s$ and $\vec{B}=\vec{B}_h+\vec{B}_s$, where $\vec{E}_h$ and $\vec{B}_h$ represent the high-frequency fields associated to the laser wave, $\vec{E}_s$ and $\vec{B}_s$ mean low frequency fields associated to the disturbance in the plasma. The contribution of the high-frequency laser wave magnetic field $\vec{B}_h$, can be neglected compared to the contribution of the laser wave high-frequency electric field, $\vec{E}_h$ as typically:$\vec{E}_h/\vec{B}_h\sim c$. and $\vec{E}_s/\vec{B}_s\sim\frac{\omega}{k}\ll 1$, The temporal dependence of $\vec{E}_h$ is supposed to be a normal mode:

$$\vec{E}_h=\vec{E}_0\mathrm{Re}[\exp(i\omega_l t)] \quad (2)$$

where $\vec{E}_0$ and ω_l are respectively the complex magnitude and the frequency of the laser wave.

In order to solve the equation (1) we consider two time scales, a low-frequency hydrodynamic time scale and high-frequency (laser field) one. Therefore, the electronic distribution function f can be written as the sum of a quasi-static distribution function f_s , which varies slowly over time and a high-frequency distribution function f_h , which follows the temporal variation of high frequency laser electric field $\vec{E}_h$, so:

$$f(\vec{r},\vec{p},t)=f_s(\vec{r},\vec{p},t)+\mathrm{Re}(f_h(\vec{p})\exp(i\omega t)) \quad (3)$$

Note that the indices "s" and "h" refer the time scales (low frequency) and high frequency respectively and will be used throughout this work.

The separation of time scales in equation (1) leads to two kinetic equations: a quasi-static kinetic equations and a high-frequency kinetic equation, so:

$$\frac{\partial f_h}{\partial t}-e\vec{E}_h.\frac{\partial f_s}{\partial \vec{p}}=e\left(\vec{E}_s+\frac{\vec{p}}{m_e\gamma_l}\times\vec{B}_s\right).\frac{\partial f_h}{\partial \vec{p}}+C_{ei}(f_h) \quad (4)$$

$$\frac{\partial f_s}{\partial t}+\frac{\vec{p}}{m_e\gamma_l}.\frac{\partial f_s}{\partial \vec{r}}-e\left(\vec{E}_s+\frac{\vec{p}}{m_e\gamma_l}\times\vec{B}_s\right).\frac{\partial f_s}{\partial \vec{p}}-C_{ei}(f_s)=\langle e\vec{E}_h.\frac{\partial f_h}{\partial \vec{p}}\rangle \quad (5)$$

where The symbol $< >$ denotes the average over the laser wave cycle time $T=2\pi/\omega_l$. The above two coupled equations are the basic equations in the present work. Note here that the terms in the electric field $\vec{E}_s$ and magnetic field, $\vec{B}_s$ reflect the inclusion, in our study, of the low frequency electromagnetic field. In particular, the first term on the right hand side in equation (4) reflects the coupling of quasi-static fields with the laser field. Let us remember here that this field present in the plasma is generated by the mechanism of the WI. The right-hand side of equation (5) is a term of beat or IB; which translates the contribution of the laser field in the description of the distribution function f_s.

3. CALCULATION OF DISTRIBUTION FUNCTION.

Firstly, we computing the high frequency distribution function from equation (4), we suppose that the effect of the quasi-static field and the (e-i) collisions are small compared to the effect of the high frequency laser field. Then it is judicious to consider the following scaling for the high frequency distribution function:

$$f_h=f_h^{(0)}(\omega_l)+f_h^{(1)}\left(\frac{v_{ei}}{\omega_l},\frac{\omega_c}{\omega_l}\right) \quad (6)$$

where the index 0 and 1 correspond to the magnitude order of high frequency distribution function. We analyze the equation (6) for typical physical parameters of laser-plasma interactions: electronic temperature, $T_e=5KeV$, the (e-i) mean free path, $\lambda_{ei}=1\mu m$ and laser wave length $\lambda_l=1.06\mu m$. It appears that the laser frequency ω_l, is very important that the collisions frequency, v_{ei} and using iterative method.

Secondly, to obtain the low frequency distribution function, we substitute the expression of the high frequency distribution function in equation (5).

For resolve this equation, we consider the following scaling:

$$f_s = f_s^{(0)} + \delta f_s \quad (7)$$

With $\delta f \ll f_s^{(0)}$,The distribution function $f_s^{(0)}$ describes the plasma in presence of high frequency laser field E_h, however δf corresponds to the perturbation associated to quasi-static electromagnetic field: E_s and B_s. The evolution equation of the perturbed function is obtained from low frequency equation by considering the first term order.

We now return to the previous analysis of Weibel modes by considering the electron ion collisions as described by the relaxation operator of Krook type, as follow:

$$C_{ei}(f_l) = \frac{-\upsilon}{p^3} m_e^3 \gamma_l^3\, l(l+1)[f_l - f_h] \quad (8)$$

Where $v = \frac{p_t^4}{2\lambda_{ei}}$ and λ_{ei} is the e-i mean free path. and l is the order of the Legender polynomial or the ordre of the component of the distribution function trucated on the Legender polynomials.

Where Now, we develop the distribution function $f_s^{(0)}(p,\mu)$. on the Legendre polynomials $p_l(\mu)$[6,7], and using the method of calculation in the reference [4], we find the equation of the distribution function,

$$\delta f_{sl\pm1} = \pm \sqrt{\frac{2\pi}{3}} \frac{eB_s}{k} \omega F_{1,1} \frac{p^3}{\gamma_l^3 m_e^3} \frac{\partial f_{s0}^{(0)}}{\partial p}$$

$$\mp \left(1 + 2vF_{1,1}\right)\gamma_l m e B_s \sqrt{\frac{2\pi}{15}}\, p_0^2 \left[\frac{1}{p^3}\frac{\partial}{\partial p}\left(p^3 \frac{\partial f_{s0}^{(0)}}{\partial p}\right) + \frac{1}{m_e^2 c^2 \gamma_l^2}\left(9 f_{s0}^{(0)} + \frac{7}{2} p \frac{\partial f_{s0}^{(0)}}{\partial p}\right)\right] \quad (9)$$

4. Relativistic weibel instability analysis

This paragraph is devoted to the analysis growth rate of the relativistic WI. We determine the dispersion relation of the Weibel modes and deduce the growth rate of WI; The dispersion relation of electromagnetic modes can be calculated using the perturbed Fokker-Planck equation coupled with Maxwell's equations presented as follows[8]:

$$\vec{\nabla} \times \vec{E_s} = -\frac{\partial \vec{B}_s}{\partial t} \quad (10)$$

and

$$\vec{\nabla} \times \vec{B_s} = -\mu_0 \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}_s}{\partial t} \quad (11)$$

where $\vec{j}$ is the current density defined by

$$\vec{j} = -i\mu_0 e \int \frac{p}{\gamma_l m_e} \delta f(\vec{r},\vec{p},t)\, \overrightarrow{dp} \quad (12)$$

By considering that the spatio-temporal dependence of the field $\vec{E}_s$ and $\vec{B_s}$ as a Fourier mode $\sim \exp\left(i\omega t - i\vec{k}.\vec{r}\right)$ equations (10) and (12) can be represented as:

$$kE_s = \omega B_s \quad (13)$$

$$kB_s = -i\mu_0 e \int \frac{p}{\gamma_l m_e}\, \delta f_s\, d\vec{p} \quad (14)$$

By developing the function δf_s, In the spherical harmonics basis $y_l^m(\theta,\varphi)$, the equation (14) reads as:

$$kB_s = -ie\mu_0 \sqrt{\frac{2\pi}{3}} \int_0^\infty \left(\frac{p}{\gamma_l m_e}\right)^3 (\delta f_{s1.-1} - \delta f_{s1.1})dp \quad (15)$$

for $f_{s0}^{(0)} = F(p)$,Where $F(p)$ is The Jüttner (relativistic Maxwellian) distribution function[9] is given by:

$$F(p) = \frac{1}{K_2(\eta)} \frac{\eta}{4\pi\,(m\,c)^3} \exp\left[-\frac{m_e c^2}{k_b T}\left(\left(1 + \frac{p^2}{m_e^2 c^2}\right)^{\frac{1}{2}} - 1\right)\right] \quad (16)$$

$\gamma_l = \left(1 + \frac{p^2}{m_e^2 c^2}\right)^{1/2}$ is the Lorentz factor and $K_2(\mu)$ denotes the modified Bessel function defined by:

$$K_2(\eta) = \frac{1}{2}(2\pi m_e k_b T)^{-3/2} e^{-\eta} \quad (17)$$

This function can be presented in the case of weakly relativistic plasma, where $\eta \gg 1$ and the modified Bessel function can be written as : $K_2(\eta) \approx \left(\frac{\pi}{2\eta}\right)^{\frac{1}{2}} \exp(-\eta)$. This approximation is justified in the inertial fusion experiments.

Typically $T_e/m_e c^2 \simeq 0.01$ for $T_e = 5\ KeV,$ By developing the relativistic Maxwellian distribution function as:

Using this equations , The dispersion relation in a plasma hight relativistic temperature, is obtained:

$$\frac{k^2c^2}{\omega_p^2} = i\frac{1}{3}\sqrt{\frac{2}{\pi}}\frac{1}{V_t^5}\omega\int_0^\infty \frac{p^7}{m_e^5\gamma_l^6}F_{1,1}\exp\left(-\frac{E}{k_b\mathrm{T}}\right)dp + \frac{\sqrt{2}}{15\sqrt{3\pi}}\frac{p_0^2}{V_t^3}\int_0^\infty \left\{\frac{p^4}{m_e^6c^2}\frac{1}{\gamma_l^3}\left[\frac{5}{2}-\frac{c^2}{V_t^2}\right]-\frac{p^2}{m_e^4\gamma_l^4}\left[\frac{9}{\gamma_l}-4\frac{c^2}{V_t^2}\right]\right\}\left(1-2vF_{1,1}\right)\exp\left(-\frac{E}{k_bT}\right)dp \quad (18)$$

The relativistic growth rate of Weibel instable mode γ is obtained explicitly from this dispersion relation, so:

$$\gamma(k) = -\frac{3}{2}\sqrt{\frac{\pi}{2}}V_t^5\frac{k^2c^2}{\omega_p^2}\frac{1}{\int_0^\infty \frac{p^7}{m_e^5\gamma_l^6}F_{1,1}\exp\left(-\frac{E}{k_bT}\right)dp} + \frac{m_e^2V_t^4}{10\sqrt{3}}\frac{p_0^2}{p_t^2}\frac{\int_0^\infty\left\{\frac{p^4}{m_e^4}\frac{1}{\gamma_l}\left[\frac{5}{2}-\frac{c^2}{V_t^2}\right]-\frac{p^2}{m_e^2\gamma_l}\left[\frac{9}{\gamma_l}-4\frac{c^2}{V_t^2}\right]\right\}\left(1-2vF_{1,1}\right)\exp\left(-\frac{E}{k_bT}\right)dp}{\int_0^\infty \frac{p^7}{m_e^5\gamma_l^6}F_{1,1}\exp\left(-\frac{E}{k_bT}\right)dp} \quad (19)$$

Where

$$F_{1,1} = \frac{1}{\left(2v+\frac{1}{30v}\frac{k^2p^8}{m_e^8\gamma_l^8}\right)}$$

5. Discussion

The first term of equation (19) corresponds to a loss term due to Landau damping and to collisions effect; it is dominated by collisions loss in the limit: $(k\lambda_{ei} \ll 1)$ while in the non-collisional limit $(k\lambda_{ei} \gg 1)$, it is dominated by the Landau damping of electromagnetic modes.

The second term, $\sim p_0^2$, corresponds to the WI source. Equation (19) gives explicitly the growth rate of the Weibel modes excited by IBA in laser fusion plasma as function of laser pulse and plasma parameters via an integral form.

The spectra of the growth rate $\gamma(k)$ which give the growth rate of the all the instables Weibel modes (not only the γ_{max}). The calculated $\gamma(k)$ in our paper, contains two contributions: a Landau damping and an instability source propotional to the second ansisotropy of the distribution function developped on the legendre polynomials, f_2 which is propotional to the laser intensity via the term $p_0^2 \sim I$.

This shows clearly that the source of the anisotropy and consequentely of the instability is the laser heating.

We have presented in (Figures 1) the growth rate spectra of WI $\gamma(k)$, as function of the collision parameter $k\lambda_{ei}$ for typical parameters of the laser pulse and plasma. We point out that without the stabilization term, $S_{IB}\left(f_{s0}^{(0)}\right)$. In addition, the comparison of the obtained spectra with previous works shows an overestimates by two orders in the growth rate of the most unstable Weibel mode in the non-relativistic case.

6. Conclusion

In conclusion, the WI is theoretically studied using the FPE by considering the Krook collisions model. The dispersion relation of the Weibel modes is explicitly established under some justified approximation in the laser-fusion experiments [10, 11]. Taking into account to stabilization effect by the inclusion of the term $S_{IB}\left(f_s^{(0)}\right)$ led to a significant reduction in the Weibel instability growth rate. Numerical treatment of model equations shows that the growth rate of the most unstable Weibel mode decreases by two orders of magnitude. This decrease in the growth rate magnitude is accompanied by a greater reduction in the spectral range of instability. For high density plasma, the Weibel modes become completely stables. Therefore, the generation of magnetic fields by the WI due to inverse bremsstrahlung should not affect the experiences of inertial confinement fusion. Several possible extension of this study is possible; namely the taking into account of the nonlinear effect [12, 13, 14] due to the high intense laser pulse.

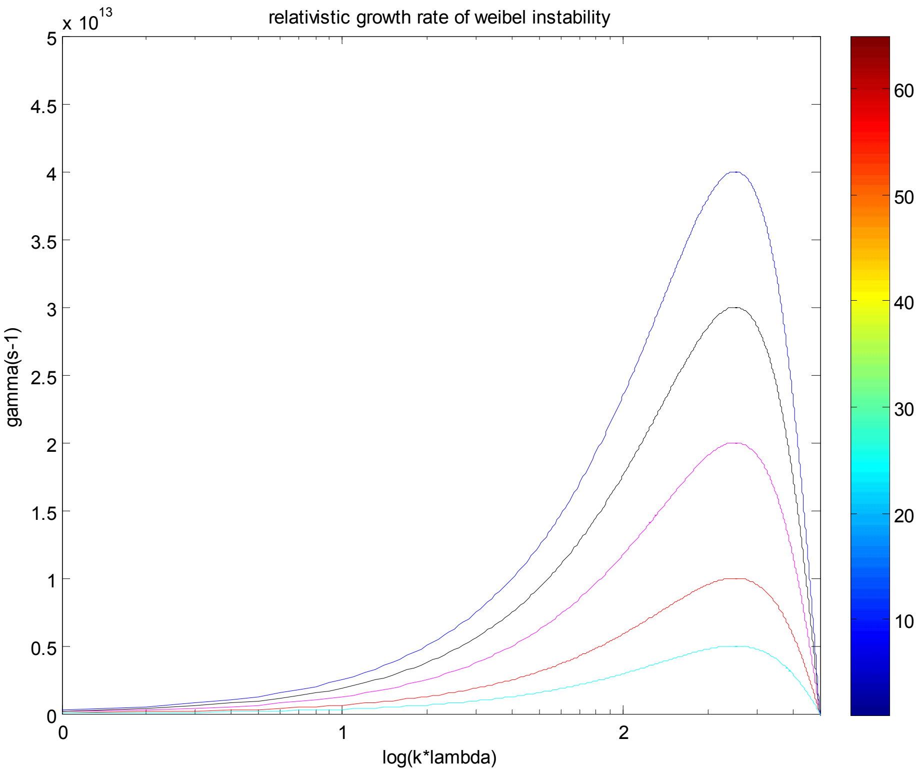

Figure 1. growth Rate of instability γ from the Krook model ,depending on the collision parameter $k\lambda_{ei}$, for typical parameters of laser pulse and fusion plasma: $T_e = 5KeV$, $\lambda_{ei} = 1\mu m$, $\lambda_l = 1.06\mu m$ $n_e=10^{26}$ cm^{-3} and $p_0/p_t=0,1$.

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